**Comparing HLM Results vs Simulations**

> #Models 1-5

> signif(gamma00table,3)

Model 1 Model 2

Bias 0.000324 0.00064

Variance 1.790000 6.99000

MSE 1.790000 6.99000

> signif(gamma01table,3)

Model 1 Model 2 Model 3 Model 4

Bias 0.0000282 0.0000558 0.0000270 0.0000534

Variance 0.0022200 0.0086700 0.0000405 0.0001580

MSE 0.0022200 0.0086700 0.0000405 0.0001580

> signif(gamma10table,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.00104 -0.00111 -0.000973 -0.000973 -0.000973

Variance 0.26500 0.51700 0.177000 0.177000 0.177000

MSE 0.26500 0.51700 0.177000 0.177000 0.177000

> signif(gamma11table,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias 0.0000366 0.0000306 0.0000369 0.0000311 0.0000428

Variance 0.0003280 0.0006400 0.0002210 0.0002260 0.0002190

MSE 0.0003280 0.0006400 0.0002210 0.0002260 0.0002190

> signif(sigmatable,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.1210 -0.0177 -0.1270 -0.0161 -0.0150

Variance 0.0186 0.0283 0.0180 0.0260 0.0241

MSE 0.0333 0.0286 0.0342 0.0262 0.0243

>

> #Models 6-10

> signif(gamma00table2,3)

Model 1 Model 2

Bias 0.00313 0.00619

Variance 1.45000 5.67000

MSE 1.45000 5.67000

> signif(gamma01table2,3)

Model 1 Model 2 Model 3 Model 4

Bias -0.000113 -0.000222 0.00000356 0.00000704

Variance 0.001810 0.007060 0.00003290 0.00012800

MSE 0.001810 0.007060 0.00003290 0.00012800

> signif(gamma10table2,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias 0.000398 -0.000273 0.00148 0.00158 0.00156

Variance 0.176000 0.294000 0.19500 0.21800 0.21800

MSE 0.176000 0.294000 0.19500 0.21800 0.21800

> signif(gamma11table2,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.00000958 0.0000145 -0.0000485 -0.0000529 -0.0000502

Variance 0.00021900 0.0003660 0.0002410 0.0002720 0.0002700

MSE 0.00021900 0.0003660 0.0002410 0.0002720 0.0002700

> signif(sigmatable2,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.1020 -0.0983 -0.1040 -0.0991 -0.0995

Variance 0.0123 0.0214 0.0122 0.0208 0.0204

MSE 0.0227 0.0311 0.0230 0.0307 0.0303

> signif(tao0table,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.2140 0.0671 -0.2270 0.0653 0.0637

Variance 0.0223 0.1080 0.0178 0.0928 0.0829

MSE 0.0681 0.1120 0.0695 0.0971 0.0870

> signif(tao1table,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias 0.04400 0.06500 0.04450 0.06520 0.06520

Variance 0.00319 0.00519 0.00315 0.00529 0.00531

MSE 0.00512 0.00941 0.00514 0.00954 0.00956

The above tables show the results of the simulations, with tables representing each of the variables in question.

We can see that, in general, the models which included the observations for I = 0 performed better than those that didn’t. For instance, consider the first table in the list:

> signif(gamma00table,3)

Model 1 Model 2

Bias 0.000324 0.00064

Variance 1.790000 6.99000

MSE 1.790000 6.99000

In the first model, we see that model 1 has about half of the bias of model 2, and has a far smaller variance and MSE. We see a similar pattern for all of the models that we have done – Model 1 and 3 perform far better than Model 2 and 4, respectively, for each estimate *except for the sigma estimate*. This indicates that including the baseline data point creates models which perform better and more accurately capture the underlying structure of the models.

However, we can see that the sigma estimate is not following this trend for the ordinary least squares models.

> signif(sigmatable,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.1210 -0.0177 -0.1270 -0.0161 -0.0150

Variance 0.0186 0.0283 0.0180 0.0260 0.0241

MSE 0.0333 0.0286 0.0342 0.0262 0.0243

Here, we see that the 2nd and 4th models have *smaller* MSE than the 1st and 3rd, respectively. Yet, they have *higher* variance. This can be explained by the bias – since both of the models produce higher bias for the sigma estimates, they actually perform more poorly than anticipated compared to the true parameters. That is, model 1 and 3 still produce better estimates when compared with their own simulated means, but perform worse when compared with the true values. Since the true values are not known in practice, this may be troublesome.

We also see that for the fixed coefficients for each model, the variance and MSE are nearly identical. This indicates that the means over all of the simulations are converging to the true means for these parameters, so taking the differences between the estimates and the mean and true values creates infinitesimally small differences in the aggregations (the variance and MSE). This is expected, and means that the models are generally performing well at estimating the true coefficients. This can be seen below:

> true

[1] 0.00 -0.20 1.00 -0.01 0.60 0.40 0.03

> signif(means,3)

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

[1,] 0.00313 -0.200 1 -0.01000 0.498 0.186 0.0740

[2,] 0.00619 -0.200 1 -0.00999 0.502 0.467 0.0950

[3,] NA -0.199 1 -0.01000 0.496 0.173 0.0745

[4,] NA -0.198 1 -0.01010 0.501 0.465 0.0952

[5,] NA -0.198 NA -0.01010 0.500 0.464 0.0952

We see that the simulations produced estimates of the coefficients whose mean is very close to the true values for all of the variables *except* the 5th, 6th and the 7th. These represent the variance estimates for both the first and second level random components (sigma, tao0 and tao1). As these numbers are more difficult to predict accurately (since randomness was introduced for each simulation), this is somewhat expected. We can see that the MSE and the variance for these estimates are *not* the same, which follows from the differences in the mean over the simulations and the true values.

We also see that, for the ordinary least squares models, the third and fourth models performed better than the first and second, respectively, as they had considerably smaller MSE and variance. This happened in every case, again, except for sigma. This indicates that taking out the intercept improves the estimate of the fixed effects – which is somewhat expected, as the true model has an intercept of 0.

However, for the random slopes models, we can see that the MSE is slightly higher in most cases than their intercept-included counterparts. This can be attributed to the fact that there is now a random part in the intercept itself. Again, we can see that the variances are smaller for these, indicating that on their own, empirical, means, these models produce better results, but when compared with the true values, they are weaker. Again, this can be attributed to the inherently added randomness to the slopes and intercepts.

In the first five models, using linear regression, subtracting the baseline from each of the values and using this as the response variable had better results than all of the other models in estimating the parameters. This is indicated by the smaller MSEs for all of the coefficients. Using this differencing method produced better results, as it was a way of standardizing – since the response variable now represents variations from the baseline, and includes the baseline as a variable, this produced better results than the others, which did not take this into account. Since these are considered to be fixed numbers in each iteration, it would be sensible that this would produce better results in the case of ordinary least squares.

However, for the random slopes models, we see an opposite effect – indeed, the fifth model performs relatively poorly in terms of the MSE. Let’s take, for example, the estimates for tao0:

> signif(tao0table,3)

Model 1 Model 2 Model 3 Model 4 Model 5

Bias -0.2140 0.0671 -0.2270 0.0653 0.0637

Variance 0.0223 0.1080 0.0178 0.0928 0.0829

MSE 0.0681 0.1120 0.0695 0.0971 0.0870

Here, we can see that the mean squared error and variance are higher in the fifth model than the 1st and 3rd models, which did not subtract the baseline. Since the intercept itself now contains a random component, assuming it to be constant and not estimating the random component is detrimental to the model’s accuracy. That is, only considering the baseline in terms of its interaction rather than by itself as well is not favorable in this example, since the intercepts are now assumed to varying with some randomness. Unlike in ordinary least squares, we cannot assume that this partial information is a fixed number.

Looking at the MSE for the similar models with each estimation shows that using the LMER method is producing better estimates for all of the variables that match in the first two models – for instance, in the case of gamma00 the MSE is significantly lower in the random slopes/intercept model than the linear squares model. However, for the 3rd-5th models, the MSE increases between the types of models.